

Student's t-test

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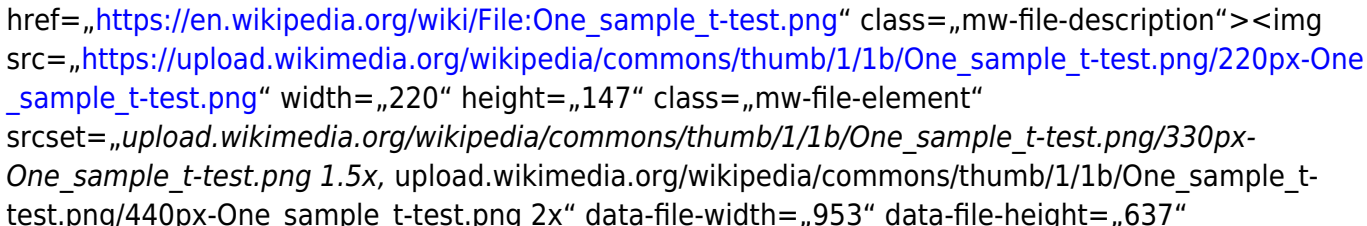
`<html> <div class=„vector-body-before-content“> </div> <div id=„mw-content-text“ class=„mw-body-content“><div class=„mw-content-ltr mw-parser-output“ lang=„en“ dir=„ltr“><p>Student's <i>t</i>-test is a statistical test used to test whether the difference between the response of two groups is statistically significant or not. It is any statistical hypothesis test in which the test statistic follows a Student's <i>t</i>-distribution under the null hypothesis. It is most commonly applied when the test statistic would follow a normal distribution if the value of a scaling term in the test statistic were known (typically, the scaling term is unknown and is therefore a nuisance parameter). When the scaling term is estimated based on the data, the test statistic—under certain conditions—follows a Student's <i>t</i> distribution. The <i>t</i>-test's most common application is to test whether the means of two populations are significantly different. In many cases, a Z-test will yield very similar results to a t-test since the latter converges to the former as the size of the dataset increases.</p><h2>History[edit]</h2><figure class=„mw-default-size“><figcaption>William Sealy Gosset, who developed the „<i>t</i>-statistic“ and published it under the pseudonym of „Student“</figcaption></figure><p>The term „<i>t</i>-statistic“ is abbreviated from „hypothesis test statistic“. ^{[1]} In statistics, the <i>t</i>-distribution was first derived as a posterior distribution in 1876 by <a href=„https://en.wikipedia.org/wiki/Friedrich_Robert_Helmert“`

[title=„Friedrich Robert Helmert“>Helmert](https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-2)^[2]
[id=„cite_ref-3“ class=„reference“>\[3\]](https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-3)^[4]
[id=„cite_ref-4“ class=„reference“>\[4\]](https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-4) and [^{\[5\]}
\[id=„cite_ref-6“ class=„reference“>\\[5\\]\]\(https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-5\)^{\[6\]}
\[id=„cite_ref-7“ class=„reference“>\\[6\\]\]\(https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-6\)^{\[7\]} The *t*-distribution also appeared in a more general form as Pearson type IV distribution in \['s 1895 paper.^{\\[8\\]} However, the *t*-distribution, also known as \\[t-distribution\\]\\(https://en.wikipedia.org/wiki/Student%27s_t-distribution "„Student's"\\), gets its name from \\[, who first published it in English in 1908 in the scientific journal \\\[using the pseudonym „Student“^{\\\\[9\\\\]}
\\\\[id=„cite_ref-The_Probable_Error_of_a_Mean_9-0“ class=„reference“>\\\\\[9\\\\\]\\\\]\\\\(https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-The_Probable_Error_of_a_Mean-9\\\\)^{\\\\[10\\\\]} because his employer preferred staff to use \\\\[when publishing scientific papers.^{\\\\\[11\\\\\]}
\\\\\[id=„cite_ref-10“ class=„reference“>\\\\\\[11\\\\\\]\\\\\]\\\\\(https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-11\\\\\)^{\\\\\[12\\\\\]} Gosset worked at the \\\\\[in \\\\\\[, \\\\\\\[, and was interested in the problems of small samples; for example, the chemical properties of barley with small sample sizes. Hence a second version of the etymology of the term Student is that Guinness did not want their competitors to know that they were using the *t*-test to determine the quality of raw material. Although it was William Gosset after whom the term „Student“ is penned, it was actually through the work of \\\\\\\\[that the distribution became well known as „Student's distribution“^{\\\\\\\\\[12\\\\\\\\\]}
\\\\\\\\\[id=„cite_ref-12“ class=„reference“>\\\\\\\\\\[12\\\\\\\\\\]\\\\\\\\\]\\\\\\\\\(https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-12\\\\\\\\\) and „Student's *t*-test“. Gosset devised the *t*-test as an economical way to monitor the quality of \\\\\\\\\[. The *t*-test work was submitted to and accepted in the journal \\\\\\\\\\[and published in 1908.^{\\\\\\\\\\\[9\\\\\\\\\\\]}
\\\\\\\\\\\[id=„cite_ref-The_Probable_Error_of_a_Mean_9-1“ class=„reference“>\\\\\\\\\\\\[9\\\\\\\\\\\\]\\\\\\\\\\\]\\\\\\\\\\\(https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-The_Probable_Error_of_a_Mean-9\\\\\\\\\\\)^{\\\\\\\\\\\[9\\\\\\\\\\\]} Guinness had a policy of allowing technical staff leave for study \\\\\\\\\\\(so-called „study leave“\\\\\\\\\\\), which Gosset used during the first two terms\\\\\\\\\\]\\\\\\\\\\(https://en.wikipedia.org/wiki/Biometrika "„Biometrika“>Biometrika</a"\\\\\\\\\\)\\\\\\\\\]\\\\\\\\\(https://en.wikipedia.org/wiki/Stout "„Stout“>stout</a"\\\\\\\\\)\\\\\\\\]\\\\\\\\(https://en.wikipedia.org/wiki/Ronald_Fisher "„Ronald"\\\\\\\\)\\\\\\\]\\\\\\\(https://en.wikipedia.org/wiki/Ireland "„Ireland“>Ireland</a"\\\\\\\)\\\\\\]\\\\\\(https://en.wikipedia.org/wiki/Dublin "„Dublin“>Dublin</a"\\\\\\)\\\\\]\\\\\(https://en.wikipedia.org/wiki/Guinness_Brewery "„Guinness"\\\\\)\\\\]\\\\(https://en.wikipedia.org/wiki/Pen_name "„Pen"\\\\)\\\]\\\(https://en.wikipedia.org/wiki/Biometrika "„Biometrika“>Biometrika</a"\\\)\\]\\(https://en.wikipedia.org/wiki/William_Sealy_Gosset "„William"\\)\]\(https://en.wikipedia.org/wiki/Karl_Pearson "„Karl"\)](https://en.wikipedia.org/wiki/Jacob_L%C3%BCroth "„Jacob")

of the 1906–1907 academic year in Professor [Karl Pearson](https://en.wikipedia.org/wiki/Karl_Pearson "Karl Pearson")'s Biometric Laboratory at [University College London](https://en.wikipedia.org/wiki/University_College_London "University College London").^[13] Gosset's identity was then known to fellow statisticians and to editor-in-chief Karl Pearson.^[14]

Uses

[edit](https://en.wikipedia.org/w/index.php?title=Student%27s_t-test&action=edit§ion=2 "Edit section: Uses")



[One sample t-test](https://upload.wikimedia.org/wikipedia/commons/thumb/1/1b/One_sample_t-test.png/220px-One_sample_t-test.png) [1.5x](https://upload.wikimedia.org/wikipedia/commons/thumb/1/1b/One_sample_t-test.png/330px-One_sample_t-test.png), [2x](https://upload.wikimedia.org/wikipedia/commons/thumb/1/1b/One_sample_t-test.png/440px-One_sample_t-test.png) [2_Sample_Test](https://en.wikipedia.org/wiki/File:2_Sample_Test.png) [1.5x](https://upload.wikimedia.org/wikipedia/commons/thumb/1/1c/2_Sample_Test.png/220px-2_Sample_Test.png), [2x](https://upload.wikimedia.org/wikipedia/commons/thumb/1/1c/2_Sample_Test.png/440px-2_Sample_Test.png)

One-sample t-test

A **one-sample Student's t-test** is a [location test](https://en.wikipedia.org/wiki/Location_test "Location test") of whether the mean of a population has a value specified in a [null hypothesis](https://en.wikipedia.org/wiki/Null_hypothesis "Null hypothesis"). In testing the null hypothesis that the population mean is equal to a specified value μ_0 , one uses the statistic

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

where $\bar{x}$ is the sample mean, s is the [sample standard deviation](https://en.wikipedia.org/wiki/Standard_deviation#Estimation "Standard deviation") and n is the sample size. The degrees of freedom used in this test are $n - 1$. Although the parent population does not need to be normally distributed, the distribution of the population of sample means $\bar{x}$ is assumed to be normal.

By the [central limit theorem](https://en.wikipedia.org/wiki/Central_limit_theorem "Central limit theorem"), if the observations are independent and the second moment exists, then

src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/65658b7b223af9e1acc877d848888ecd b4466560“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„ $\{ \displaystyle t \}$ “ referrerpolicy=„no-referrer“ /> will be approximately normal .</p><h3>Two-sample <i>t</i>-tests</h3>edit</h3><figure class=„mw-default-size“><figcaption>Type I error of unpaired and paired two-sample <i>t</i>-tests as a function of the correlation. The simulated random numbers originate from a bivariate normal distribution with a variance of 1. The significance level is 5% and the number of cases is 60.</figcaption></figure><figure class=„mw-default-size“><figcaption>Power of unpaired and paired two-sample <i>t</i>-tests as a function of the correlation. The simulated random numbers originate from a bivariate normal distribution with a variance of 1 and a deviation of the expected value of 0.4. The significance level is 5% and the number of cases is 60.</figcaption></figure><p>A two-sample location test of the null hypothesis such that the means of two populations are equal. All such tests are usually called Student's <i>t</i>-tests, though strictly speaking that name should only be used if the <a href=„<https://en.wikipedia.org/wiki/Variance>“ title=„Variance“>variances of the two populations are also assumed to be equal; the form of the test used when this assumption is dropped is sometimes called Welch's <i>t</i>-test. These tests are often referred to as unpaired or <i>independent samples</i> <i>t</i>-tests, as they are typically applied when the statistical units underlying the two samples being compared are non-overlapping.^{[15]}</p><p>Two-sample <i>t</i>-tests for a difference in means involve independent samples (unpaired samples) or paired samples. Paired <i>t</i>-tests are a form of blocking, and have greater power (probability of avoiding a type II error, also known as a false negative) than unpaired tests when the paired units are similar with respect to „noise factors“ (see <a

href=„<https://en.wikipedia.org/wiki/Confounder>“ class=„mw-redirect“ title=„Confounder“>confounder) that are independent of membership in the two groups being compared.^{[16]} In a different context, paired <i>t</i>-tests can be used to reduce the effects of <a href=„<https://en.wikipedia.org/wiki/Confounders>“ class=„mw-redirect“ title=„Confounders“>confounding factors in an observational study.</p><h4>Independent (unpaired) samples^{edit}</h4><p>The independent samples <i>t</i>-test is used when two separate sets of independent and identically distributed samples are obtained, and one variable from each of the two populations is compared. For example, suppose we are evaluating the effect of a medical treatment, and we enroll 100 subjects into our study, then randomly assign 50 subjects to the treatment group and 50 subjects to the control group. In this case, we have two independent samples and would use the unpaired form of the <i>t</i>-test.</p><h4>Paired samples^{edit}</h4><div data-mw-deduplicate=„TemplateStyles:r1033289096“ data-mw-scoped=„scoped“><div role=„note“ class=„hatnote navigation-not-searchable“>Main article: Paired difference test</div><p>Paired samples <i>t</i>-tests typically consist of a sample of matched pairs of similar units, or one group of units that has been tested twice (a „repeated measures“ <i>t</i>-test).</p><p>A typical example of the repeated measures <i>t</i>-test would be where subjects are tested prior to a treatment, say for high blood pressure, and the same subjects are tested again after treatment with a blood-pressure-lowering medication. By comparing the same patient's numbers before and after treatment, we are effectively using each patient as their own control. That way the correct rejection of the null hypothesis (here: of no difference made by the treatment) can become much more likely, with statistical power increasing simply because the random interpatient variation has now been eliminated. However, an increase of statistical power comes at a price: more tests are required, each subject having to be tested twice. Because half of the sample now depends on the other half, the paired version of Student's <i>t</i>-test has only $n/2 - 1$ degrees of freedom (with <i>n</i> being the total number of observations). Pairs become individual test units, and the sample has to be doubled to achieve the same number of degrees of freedom. Normally, there are $n - 1$ degrees of freedom (with <i>n</i> being the total number of observations).^{[17]}<p>A paired samples <i>t</i>-test based on a „matched-pairs sample“ results from an unpaired sample that is subsequently used to form a paired sample, by using additional variables that were measured along with the variable of interest.^{[18]} The matching is carried out by identifying pairs of values consisting of one observation from each of the two samples, where the pair is similar in terms of other measured variables. This approach is sometimes used in observational studies to reduce or eliminate the effects of confounding factors.</p><p>Paired samples <i>t</i>-tests are often referred to as

„dependent samples t -tests“.

Assumptions

Most test statistics have the form $t = \frac{Z}{s}$, where Z and s are functions of the data. Z may be sensitive to the alternative hypothesis (i.e., its magnitude tends to be larger when the alternative hypothesis is true), whereas s is a scaling parameter that allows the distribution of t to be determined. As an example, in the one-sample t -test

$$t = \frac{\bar{X} - \mu}{\hat{\sigma} / \sqrt{n}}$$

where $\bar{X}$ is the sample mean from a sample $X_1, X_2, \dots, X_n$ of size n , s is the standard error of the mean, $\hat{\sigma} = \sqrt{\frac{1}{n-1} \sum (X_i - \bar{X})^2}$ is the estimate of the standard deviation of the population, and σ^2 is the population variance. The assumptions underlying a t -test in the simplest form above are that:

- X follows a normal distribution with mean μ and variance σ^2 .
- s^2 follows a χ^2 distribution with $n-1$ degrees of freedom (statistics).

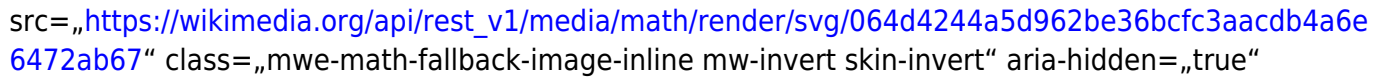
This assumption is met when the observations used for estimating s^2 come from a normal distribution (and X is i.i.d. for each group).
- Z and s are statistically independent.

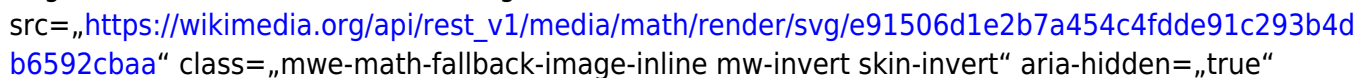
In the t -test comparing the means of two independent samples, the following assumptions should be met:

- The means of the two populations being compared should follow normal distributions. Under weak assumptions, this follows in large samples from the central limit theorem, even when the distribution of observations in each group is non-normal.
- If using Student's original definition of the t -test, the two populations being compared should have the same variance (testable using an F -test of equality of variances).

title=„Levene's test">Levene's test, Bartlett's test, or the Brown–Forsythe test; or assessable graphically using a Q–Q plot). If the sample sizes in the two groups being compared are equal, Student's original <i>t</i>-test is highly robust to the presence of unequal variances.^{[20]}Welch's <i>t</i>-test is insensitive to equality of the variances regardless of whether the sample sizes are similar.The data used to carry out the test should either be sampled independently from the two populations being compared or be fully paired. This is in general not testable from the data, but if the data are known to be dependent (e.g. paired by test design), a dependent test has to be applied. For partially paired data, the classical independent <i>t</i>-tests may give invalid results as the test statistic might not follow a <i>t</i>-distribution, while the dependent <i>t</i>-test is sub-optimal as it discards the unpaired data.^{[21]}<p>Most two-sample <i>t</i>-tests are robust to all but large deviations from the assumptions.^{[22]}</p><p>For exactness, the <i>t</i>-test and <i>Z</i>-test require normality of the sample means, and the <i>t</i>-test additionally requires that the sample variance follows a scaled <i> χ^2 </i>-distribution, and that the sample mean and sample variance be statistically independent. Normality of the individual data values is not required if these conditions are met. By the central limit theorem, sample means of moderately large samples are often well-approximated by a normal distribution even if the data are not normally distributed. However, the sample size required for the sample means to converge to normality depends on the skewness of the distribution of the original data. The sample can vary from 30 to 100 or higher values depending on the skewness.^{[23]}^{[24]}</p><p>For non-normal data, the distribution of the sample variance may deviate substantially from a <i> χ^2 </i>-distribution.</p><p>However, if the sample size is large, Slutsky's theorem implies that the distribution of the sample variance has little effect on the distribution of the test statistic. That is, as sample size increases:</p><dl><dd></dd></dl><p>as per the Central limit

theorem

 class="mwe-math-fallback-image-inline mw-invert skin-invert" aria-hidden="true" alt=",{\displaystyle s^2\rightarrow p \sigma ^2}" referrerpolicy="no-referrer" /> as per the [law of large numbers](https://en.wikipedia.org/wiki/Law_of_large_numbers)

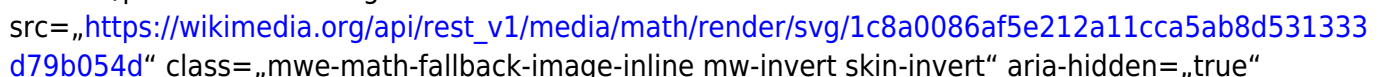
 class="mwe-math-fallback-image-inline mw-invert skin-invert" aria-hidden="true" alt=",{\displaystyle \text{therefore } {\frac {\sqrt {n}}{s}}(\bar {X}-\mu)\rightarrow d N(0,1)}" referrerpolicy="no-referrer" />.

[edit](https://en.wikipedia.org/w/index.php?title=Student%27s_t-test&action=edit§ion=8)

Explicit expressions that can be used to carry out various *t*-tests are given below. In each case, the formula for a test statistic that either exactly follows or closely approximates a *t*-distribution under the null hypothesis is given. Also, the appropriate [degrees of freedom](https://en.wikipedia.org/wiki/Degrees_of_freedom_(statistics)) are given in each case. Each of these statistics can be used to carry out either a [one-tailed or two-tailed test](https://en.wikipedia.org/wiki/One-tailed_test). Once the *t* value and degrees of freedom are determined, a [*p*-value](https://en.wikipedia.org/wiki/P-value) can be found using a [table of values from Student's *t*-distribution](https://en.wikipedia.org/wiki/Student%27s_t-distribution#Table_of_selected_values). If the calculated *p*-value is below the threshold chosen for [statistical significance](https://en.wikipedia.org/wiki/Statistical_significance) (usually the 0.10, the 0.05, or 0.01 level), then the null hypothesis is rejected in favor of the alternative hypothesis.

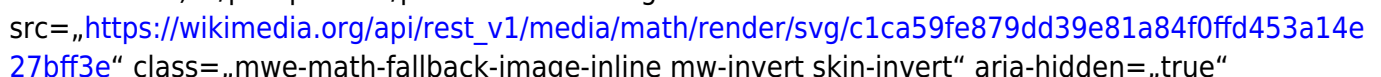
[edit](https://en.wikipedia.org/w/index.php?title=Student%27s_t-test&action=edit§ion=9)

Suppose one is fitting the model

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where *x* is known, *ε* and *Y* are unknown, *ε* is a normally distributed random variable with mean 0 and unknown variance *σ²*, and *Y* is the outcome of interest. We want to test the null hypothesis that the slope *β* is equal to some specified value *β₀* (often taken to be 0, in which case the null hypothesis is that *x* and *y* are uncorrelated).

Let

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Then

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has a *t*-distribution with *n* - 2 degrees of freedom if the null hypothesis is true. The [Simple](https://en.wikipedia.org/wiki/Simple_linear_regression#Normality_assumption)

linear regression" > standard error of the slope coefficient : </p><dl><dd><img src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/a1081873b3ddcc69337c980bdc24732781ea2f85“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„
$$SE_{\hat{\beta}} = \frac{\sqrt{\sum_{i=1}^n (y_i - \hat{y}_i)^2}}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}$$
“ referrerpolicy=„no-referrer“ /></dd></dl><p>can be written in terms of the residuals. Let </p><dl><dd><img src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/1575fa23f185305b9707f6919b0d5359d8158a87“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„
$$\begin{aligned} \hat{\varepsilon}_i &= y_i - \hat{y}_i \\ (\hat{\alpha} + \hat{\beta} x_i) &= \text{residuals} = \text{estimated errors} \\ \text{SSR} &= \sum_{i=1}^n \hat{\varepsilon}_i^2 = \text{sum of squares of residuals} \end{aligned}$$
“ referrerpolicy=„no-referrer“ /></dd></dl><p>Then t_{score} is given by </p><dl><dd><img src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/5cb5f3a4e467a67d1dfc346ca722edbb2a41145“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„
$$t_{\text{score}} = \frac{(\hat{\beta} - \beta_0) \sqrt{n-2}}{\sqrt{\text{SSR} \sum_{i=1}^n (x_i - \bar{x})^2}}$$
“ referrerpolicy=„no-referrer“ /></dd></dl><p>Another way to determine the t_{score} is </p><dl><dd><img src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/0b352cb2142064ce27ba30161b1faeb355021676“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„
$$t_{\text{score}} = \frac{r \sqrt{n-2}}{\sqrt{1-r^2}}$$
“ referrerpolicy=„no-referrer“ /></dd></dl><p>where r is the Pearson correlation coefficient. </p><p>The $t_{\text{score, intercept}}$ can be determined from the $t_{\text{score, slope}}$: </p><dl><dd><img src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/1641c084c6122f4dab5ab3ec4e9b8cb6adb8a950“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„
$$t_{\text{score, intercept}} = \frac{\hat{\alpha} \hat{\beta}}{t_{\text{score, slope}} \sqrt{s_x^2 + \bar{x}^2}}$$
“ referrerpolicy=„no-referrer“ /></dd></dl><p>where s_x^2 is the sample variance. </p><h3>Independent two-sample t-test</h3>edit</h3><h4>Equal sample sizes and variance</h4>edit</h4><p>Given two groups (1, 2), this test is only applicable when: </p>the two sample sizes are equal,it can be assumed that the two distributions have the same variance.<p>Violations of these assumptions are discussed below. </p><p>The t statistic to test whether the means are different can be calculated as follows: </p><dl><dd><img src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/09aab90a09d5dba053cb4e398f9999738c8ca5e3“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„
$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{2}{n}}}$$
“ referrerpolicy=„no-referrer“ /></dd></dl><p>where </p><dl><dd><img src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/2adbe1794944e2924c113eacd348126a39746e59“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„
$$s_p = \sqrt{\frac{s_{X1}^2 + s_{X2}^2}{2}}$$
“ referrerpolicy=„no-referrer“ /></dd></dl><p>Here s_p is the pooled standard deviation for $n < n_1 = n_2$, and

$\frac{s_1^2 + s_2^2}{2}$ and $\frac{s_1^2 + s_2^2}{n_1 + n_2 - 2}$ are the unbiased estimators of the population variance. The denominator of t is the standard error of the difference between two means.

For significance testing, the degrees of freedom for this test is $2n - 2$, where n is sample size.

Equal or unequal sample sizes, similar variances

$\frac{1}{2} \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)$

$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{1}{2} \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)}}$

This test is used only when it can be assumed that the two distributions have the same variance (when this assumption is violated, see below). The previous formulae are a special case of the formulae below, one recovers them when both samples are equal in size:

$n_1 = n_2 = n$

The t statistic to test whether the means are different can be calculated as follows:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{1}{n} \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)}}$$

where

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

is the pooled standard deviation of the two samples: it is defined in this way so that its square is an unbiased estimator of the common variance, whether or not the population means are the same. In these formulae, n_1 is the number of degrees of freedom for each group, and the total sample size minus two (that is, $n_1 + n_2 - 2$) is the total number of degrees of freedom, which is used in significance testing.

Equal or unequal sample sizes, unequal variances

$\frac{s_1^2}{n_1} > \frac{s_2^2}{n_2}$ or $\frac{s_1^2}{n_1} < \frac{s_2^2}{n_2}$

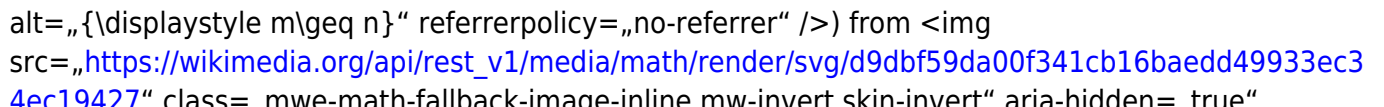
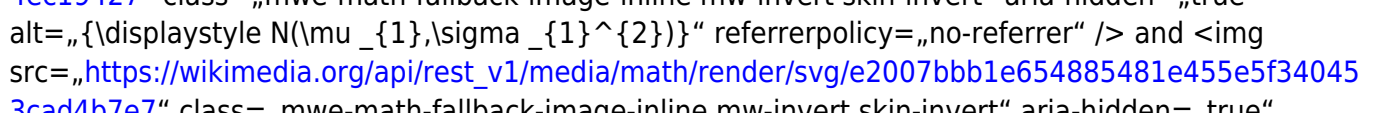
$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

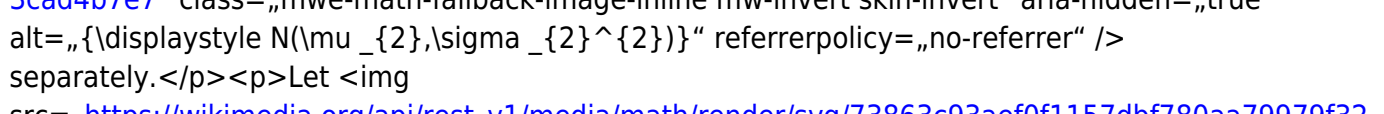
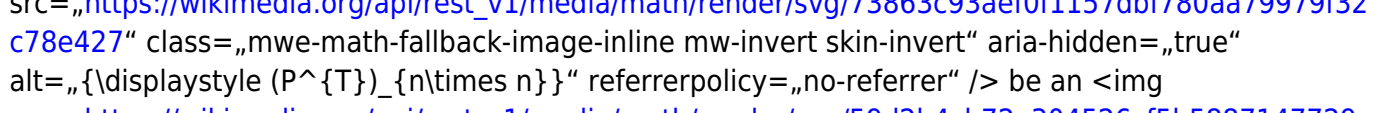
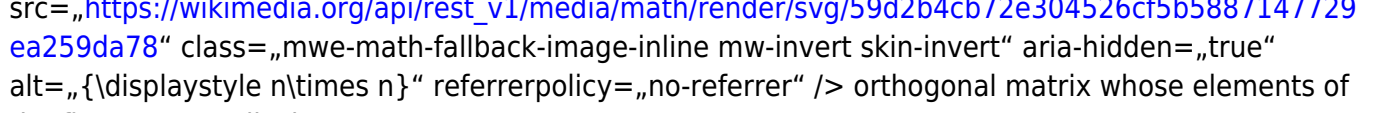
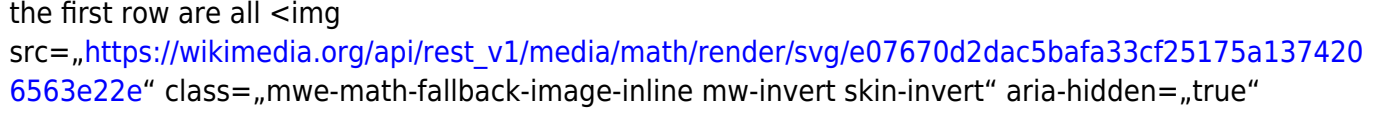
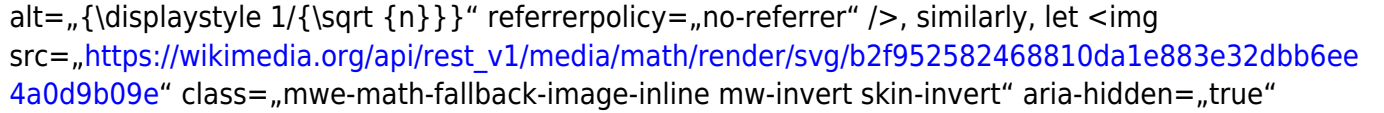
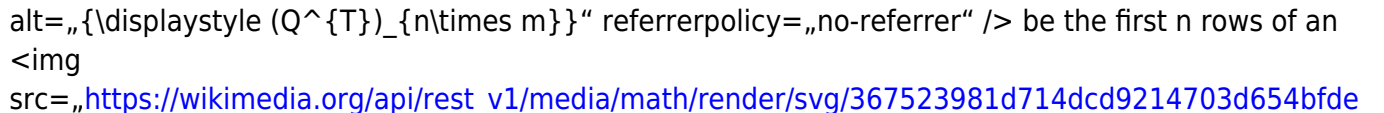
Main article: Welch's t -test

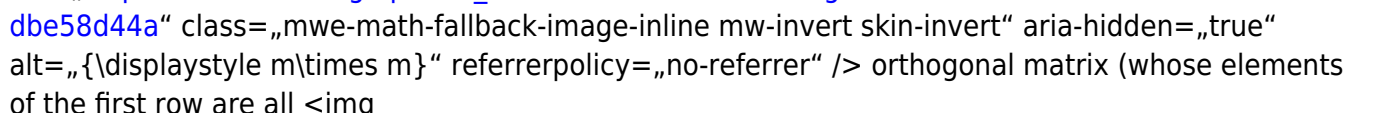
This test, also known as Welch's t -test, is used only when the two population variances are not assumed to be equal (the two sample sizes may or may not be equal) and hence must be estimated separately. The t statistic to test whether the population means are different is calculated

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

referrer" /></dd></dl><p>where</p><dl><dd><img
src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/5024dd38e905d60353a324af2a05058fdd4ac3e7“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“
alt=„ $\{\displaystyle s_{\bar{\Delta}}=\sqrt{\frac{s_1}{n_1}}+\frac{s_2^2}{n_2}\}$ “ referrerpolicy=„no-referrer“ /></dd></dl><p>Here s_i is the
unbiased estimator of the <a href=„<https://en.wikipedia.org/wiki/Variance>“
title=„Variance“>variance of each of the two samples with n_i = number of participants
in group i ($i = 1$ or 2). In this case <img
src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/14261fb3a8ce72b3cb026b279ecab8452714fc85“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“
alt=„ $\{\displaystyle (s_{\Delta})^2$ “ referrerpolicy=„no-referrer“ />is not a pooled
variance. For use in significance testing, the distribution of the test statistic is approximated as an
ordinary Student's t -distribution with the degrees of freedom calculated
using</p><dl><dd><img
src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/25159fe546cf459632f4743d874de9d0960cdc16“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“
alt=„ $\{\displaystyle \text{d.f.}=\frac{\left(\frac{s_1}{n_1}\right)^2}{\frac{s_1}{n_1}+\frac{s_2^2}{n_2}}+\frac{(s_2^2/n_2)^2}{(s_2^2/n_2)+1}\}$ “ referrerpolicy=„no-referrer“ /></dd></dl><p>This is
known as the <a href=„https://en.wikipedia.org/wiki/Welch%E2%80%93Satterthwaite_equation“
title=„Welch–Satterthwaite equation“>Welch–Satterthwaite equation. The true
distribution of the test statistic actually depends (slightly) on the two unknown population variances
(see <a href=„https://en.wikipedia.org/wiki/Behrens%E2%80%93Fisher_problem“
title=„Behrens–Fisher problem“>Behrens–Fisher problem).</p><h3>Exact
method for unequal variances and sample sizes[<a
href=„https://en.wikipedia.org/w/index.php?title=Student%27s_t-test&action=edit§ion=14“ title=„Edit section: Exact method for unequal variances and sample
sizes“>edit]</h3><p>The test<sup id=„cite_ref-25“ class=„reference“>[25]</sup> deals with
the famous <a href=„https://en.wikipedia.org/wiki/Behrens%E2%80%93Fisher_problem“
title=„Behrens–Fisher problem“>Behrens–Fisher problem, i.e., comparing the
difference between the means of two normally distributed populations when the variances of the two
populations are not assumed to be equal, based on two independent samples.</p><p>The test is
developed as an exact test
that allows for unequal sample sizes and unequal variances of two populations.
The exact property still holds even with small extremely small and unbalanced sample sizes
(e.g. <img
src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/1c17d0c5c824726f2b3ffd5ea3200fe1636982eb“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“
alt=„ $n_1=5, n_2=50$ “ referrerpolicy=„no-referrer“ />).</p><p>The statistic to
test whether the means are different can be calculated as follows:</p><p>Let <img
src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/a0f37f25b326e4b6229a7f0be5283ace07d1a97f“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“
alt=„ $X=[X_1, X_2, \ldots, X_m]^T$ “ referrerpolicy=„no-referrer“ /> and <img
src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/a81b49d1f74f1a3c22759407966c63524eac1d2e“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“
alt=„ $Y=[Y_1, Y_2, \ldots, Y_n]^T$ “ referrerpolicy=„no-referrer“ /> be the i.i.d.
sample vectors (<img
src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/6b02f25e62da7fe3162ac80446437cdc1c0fd341“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“

alt=„ $\{ \displaystyle m \geq n \}$ “ referrerpolicy=„no-referrer“ />) from  src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/d9dbf59da00f341cb16baedd49933ec34ec19427“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„ $\{ \displaystyle N(\mu_1, \sigma_1^2) \}$ “ referrerpolicy=„no-referrer“ /> and  src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/e2007bbb1e654885481e455e5f340453cad4b7e7“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„ $\{ \displaystyle N(\mu_2, \sigma_2^2) \}$ “ referrerpolicy=„no-referrer“ /> separately.

Let  src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/73863c93aef0f1157dbf780aa79979f32c78e427“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„ $\{ \displaystyle (P^T)_n \}$ “ referrerpolicy=„no-referrer“ /> be an  src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/59d2b4cb72e304526cf5b5887147729ea259da78“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„ $\{ \displaystyle n \}$ “ referrerpolicy=„no-referrer“ /> orthogonal matrix whose elements of the first row are all  src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/e07670d2dac5bafa33cf25175a1374206563e22e“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„ $\{ \displaystyle 1/\sqrt{n} \}$ “ referrerpolicy=„no-referrer“ />, similarly, let  src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/b2f952582468810da1e883e32dbb6ee4a0d9b09e“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„ $\{ \displaystyle (Q^T)_n \}$ “ referrerpolicy=„no-referrer“ /> be the first n rows of an  src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/367523981d714dcd9214703d654bfdebbe58d44a“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„ $\{ \displaystyle m \}$ “ referrerpolicy=„no-referrer“ /> orthogonal matrix (whose elements of the first row are all  src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/b9d0ccf6bf7c8456f1b832ad59ec1d717380cd68“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„ $\{ \displaystyle 1/\sqrt{m} \}$ “ referrerpolicy=„no-referrer“ />).

Then  src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/53fdf3e89b640dbb4ea79722739b9947771cdf06“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„ $\{ \displaystyle Z=(Q^T)_n X/\sqrt{m}-(P^T)_n Y/\sqrt{n} \}$ “ referrerpolicy=„no-referrer“ /> is an n-dimensional normal random vector.

$Z \sim N^{1)}$

Perform the t-test. Notice that the assumption of equal variance,

```
var.equal=T
```

, is required to make the analysis exactly equivalent to simple linear regression.

```
dir="ltr"><pre>>with(word.recall.data,t.test(word.recall~drug.dose,var.equal=T))</pre></div><p>Running the R code gives the following results.</p><ul><li>The mean word.recall in the 0 drug.dose group is 2.</li><li>The mean word.recall in the 1 drug.dose group is 6.</li><li>The difference between treatment groups in the mean word.recall is 6 &#8211; 2 = 4.</li><li>The difference in word.recall between drug doses is significant (p=0.00805).</li></ul><p>Perform a linear regression of the same data. Calculations may be performed using the R function
```

```
lm()
```

for a linear model.

```
>word.recall.data.lm=lm(word.recall~drug.dose,data=word.recall.data)>summary(word.recall.data.lm)
```

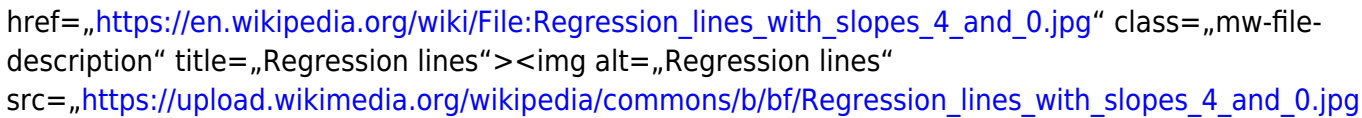
 The linear regression provides a table of coefficients and p-values.

Coefficient	Estimate	Std. Error	t value	P-value
Intercept	2	0.5774	3.464	0.02572
drug.dose	4	0.8165	4.899	0.00805

The table of coefficients gives the following results.

- The estimate value of 2 for the intercept is the mean value of the word recall when the drug dose is 0.
- The estimate value of 4 for the drug dose indicates that for a 1-unit change in drug dose (from 0 to 1) there is a 4-unit change in mean word recall (from 2 to 6). This is the slope of the line joining the two group means.
- The p-value that the slope of 4 is different from 0 is $p = 0.00805$.

The coefficients for the linear regression specify the slope and intercept of the line that joins the two group means, as illustrated in the graph. The intercept is 2 and the slope is 4.



Compare the result from the linear regression to the result from the t-test.

- From the t-test, the difference between the group means is $6 - 2 = 4$.
- From the regression, the slope is also 4 indicating that a 1-unit change in drug dose (from 0 to 1) gives a 4-unit change in mean word recall (from 2 to 6).
- The t-test p-value for the difference in means, and the regression p-value for the slope, are both 0.00805. The methods give identical results.

This example shows that, for the special case of a simple linear regression where there is a single x-variable that has values 0 and 1, the t-test gives the same results as the linear regression. The relationship can also be shown algebraically.

Recognizing this relationship between the t-test and linear regression facilitates the use of multiple linear regression and multi-way analysis of variance. These alternatives to t-tests allow for the inclusion of additional explanatory variables that are associated with the response. Including such additional explanatory variables using regression or anova reduces the otherwise unexplained variance, and commonly yields greater power to detect differences than do two-sample t-tests.

Software implementations

Many spreadsheet programs and statistics packages, such as QtiPlot, LibreOffice Calc, Microsoft Excel, SAS (software), SPSS, Stata, DAP (software), Gretl, R (programming

language)">R, Python, PSPP, Wolfram Mathematica, MATLAB and Minitab, include implementations of Student's *t*-test.

Language/Program	Function	Notes
Microsoft Excel pre 2010		

TTEST(*array1*, *array2*, *tails*, *type*)

See [1]

Microsoft Excel 2010 and later

T.TEST(*array1*, *array2*, *tails*, *type*)

See [2]

Apple Numbers

TTEST(sample-1-values, sample-2-values, tails, test-type)

See [3]

LibreOffice Calc

TTEST(*Data1*; *Data2*; *Mode*; *Type*)

See [4]

Google Sheets

TTEST(range1, range2, tails, type)

See [5]

Python

scipy.stats.ttest_ind(*a*, *b*, *equal_var*=True)

See [6]

`<tr><tr><td><a href=„https://en.wikipedia.org/wiki/MATLAB“ title=„MATLAB“>MATLAB</a></td><td>`

`ttest(data1, data2)`

`</td><td>See <a rel=„nofollow“ class=„external autonumber“ href=„http://www.mathworks.com/help/stats/ttest.html“>[7]</a></td></tr><tr><td><a href=„https://en.wikipedia.org/wiki/Mathematica“ class=„mw-redirect“ title=„Mathematica“>Mathematica</a></td><td>`

`TTest[{data1, data2}]`

`</td><td>See <a rel=„nofollow“ class=„external autonumber“ href=„https://reference.wolfram.com/language/ref/TTest.html“>[8]</a></td></tr><tr><td><a href=„https://en.wikipedia.org/wiki/R\_\(programming\_language\)“ title=„R (programming language)“>R</a></td><td>`

`t.test(data1, data2, var.equal=TRUE)`

`</td><td>See <a rel=„nofollow“ class=„external autonumber“ href=„https://stat.ethz.ch/R-manual/R-devel/library/stats/html/t.test.html“>[9]</a></td></tr><tr><td><a href=„https://en.wikipedia.org/wiki/SAS\_\(software\)“ title=„SAS (software)“>SAS</a></td><td>`

`PROC TTEST`

`</td><td>See [10]</td></tr><tr><td>Java</td><td>`

`tTest(sample1, sample2)`

`</td><td>See [11]</td></tr><tr><td>Julia</td><td>`

`EqualVarianceTTest(sample1, sample2)`

`</td><td>See <a rel=„nofollow“ class=„external autonumber“ href=„https://juliastats.org/HypothesisTests.jl/stable/parametric/#HypothesisTests.EqualVarianceTTest“>[12]</a></td></tr><tr><td><a href=„https://en.wikipedia.org/wiki/Stata“ title=„Stata“>Stata</a></td><td>`

`ttest data1 == data2`

`</td><td>See <a rel=„nofollow“ class=„external autonumber“`

href=„<https://www.stata.com/manuals/rtest.pdf>“>[13]</td></tr></tbody></table><h2>See also[edit]</h2><style data-mw-deduplicate=„TemplateStyles:r1214689105“ scoped=„scoped“></style>Conditional change model<a href=„<https://en.wikipedia.org/wiki/F-test>“ title=„F-test“><i>F</i>-test – Statistical hypothesis test, mostly using multiple restrictionsNoncentral <i>t</i>-distribution in power analysis – Probability distributionStudent's <i>t</i>-statistic – Ratio in statisticsPages displaying short descriptions of redirect targets<a href=„<https://en.wikipedia.org/wiki/Z-test>“ title=„Z-test“><i>Z</i>-test – Statistical testMann–Whitney <i>U</i> test – Nonparametric test of the null hypothesisŠidák correction for <i>t</i>-test – Statistical methodWelch's <i>t</i>-test – Statistical test of whether two populations have equal meansAnalysis of variance – Collection of statistical models (ANOVA)<h2>References[edit]</h2><style data-mw-deduplicate=„TemplateStyles:r1217336898“ scoped=„scoped“></style><div class=„reflist“><div class=„mw-references-wrap mw-references-columns“><ol class=„references“><li id=„cite_note-1“>^ <style data-mw-deduplicate=„TemplateStyles:r1215172403“ scoped=„scoped“></style><cite class=„citation book cs1“><a rel=„nofollow“ class=„external text“ href=„<https://books.google.com/books?id=kiToDwAAQBAJ&pg=PA397>“><i>The Microbiome in Health and Disease</i>. Academic Press. 2020-05-29. p. 397. ISBN <a href=„<https://en.wikipedia.org/wiki/Special:BookSources/978-0-12-820001-8>“ title=„Special:BookSources/978-0-12-820001-8“><bdi>978-0-12-820001-8</bdi>.</cite><li id=„cite_note-2“>^ <link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1215172403“ /><cite id=„CITEREFSzabó2003“ class=„citation book cs1 cs1-prop-foreign-lang-source“>Szabó, István (2003). „Systeme aus einer endlichen Anzahl starrer Körper“. <i>Einführung in die Technische Mechanik</i> (in German). Springer Berlin Heidelberg. pp. 196–199. doi:10.1007/978-3-642-61925-0_16. ISBN <a href=„<https://en.wikipedia.org/wiki/Special:BookSources/978-3-540-13293-6>“ title=„Special:BookSources/978-3-540-13293-6“><bdi>978-3-540-13293-6</bdi>.</cite>

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href=„<https://mathscinet.ams.org/mathscinet-getitem?mr=1766040>“>1766040.</cite><li id=„cite_note-7“>^ <link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1215172403“ /><cite id=„CITEREFSheynin1995“ class=„citation journal cs1“>Sheynin, Oscar (1995). „Helmert's work in the theory of errors“. <i>Archive for History of Exact Sciences</i>. 49 (1): 73–104. doi:<a rel=„nofollow“ class=„external text“ href=„<https://doi.org/10.1007%2FBF00374700>“>10.1007/BF00374700. ISSN <a rel=„nofollow“ class=„external text“ href=„<https://www.worldcat.org/issn/0003-9519>“>0003-9519. S2CID <a rel=„nofollow“ class=„external text“ href=„<https://api.semanticscholar.org/CorpusID:121241599>“>121241599.</cite><li id=„cite_note-8“>^ <link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1215172403“ /><cite id=„CITEREFPearson1895“ class=„citation journal cs1“>Pearson, Karl (1895). <a rel=„nofollow“ class=„external text“ href=„<https://doi.org/10.1098%2Frsta.1895.0010>“>„X. Contributions to the mathematical theory of evolution.—II. Skew variation in homogeneous material“. <i>Philosophical Transactions of the Royal Society of London A</i>. 186: 343–414. Bibcode:<a rel=„nofollow“ class=„external text“ href=„<https://ui.adsabs.harvard.edu/abs/1895RSPTA.186..343P>“>1895RSPTA.186..343P. doi:<a rel=„nofollow“ class=„external text“ href=„<https://doi.org/10.1098%2Frsta.1895.0010>“>10.1098/rsta.1895.0010.</cite><li id=„cite_note-The_Probable_Error_of_a_Mean-9“>^ <i>a</i> <i>b</i> <link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1215172403“ /><cite id=„CITEREFStudent1908“ class=„citation journal cs1“>Student (1908). „The Probable Error of a Mean“ (PDF). <i><a href=„<https://en.wikipedia.org/wiki/Biometrika>“ title=„Biometrika“>Biometrika</i>. 6 (1): 1–25. doi:<a rel=„nofollow“ class=„external text“ href=„<https://doi.org/10.1093%2Fbiomet%2F6.1.1>“>10.1093/biomet/6.1.1. hdl:<a rel=„nofollow“ class=„external text“ href=„<https://hdl.handle.net/10338.dmlcz%2F143545>“>10338.dmlcz/143545. Retrieved 24 July 2016.</cite><li id=„cite_note-10“>^ <link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1215172403“ /><cite class=„citation web cs1“><a rel=„nofollow“ class=„external text“ href=„<https://www.tdistributiontable.com>“>„T Table“.</cite><li id=„cite_note-11“>^ <link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1215172403“ /><cite

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href=„<https://doi.org/10.2307%2F2684360>“>10.2307/2684360. JSTOR <a rel=„nofollow“ class=„external text“ href=„<https://www.jstor.org/stable/2684360>“>2684360.</cite><li id=„cite_note-Guo2017-21“>^ <link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1215172403“ /><cite id=„CITEREFGuoYuan2017“ class=„citation journal cs1“>Guo, Beibei; Yuan, Ying (2017). „A comparative review of methods for comparing means using partially paired data“. <i>Statistical Methods in Medical Research</i>. 26 (3): 1323–1340. doi:<a rel=„nofollow“ class=„external text“ href=„<https://doi.org/10.1177%2F0962280215577111>“>10.1177/0962280215577111. PMID <a rel=„nofollow“ class=„external text“ href=„<https://pubmed.ncbi.nlm.nih.gov/25834090>“>25834090. S2CID <a rel=„nofollow“ class=„external text“ href=„<https://api.semanticscholar.org/CorpusID:46598415>“>46598415.</cite><li id=„cite_note-Bland1995-22“>^ <link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1215172403“ /><cite id=„CITEREFBland1995“ class=„citation book cs1“>Bland, Martin (1995). <a rel=„nofollow“ class=„external text“ href=„<https://books.google.com/books?id=v6xpAAAMAAJ>“><i>An Introduction to Medical Statistics</i>. Oxford University Press. p. 168. ISBN <a href=„<https://en.wikipedia.org/wiki/Special:BookSources/978-0-19-262428-4>“ title=„Special:BookSources/978-0-19-262428-4“><bdi>978-0-19-262428-4</bdi>.</cite><li id=„cite_note-23“>^ <link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1215172403“ /><cite class=„citation web cs1“><a rel=„nofollow“ class=„external text“ href=„<https://analyse-it.com/docs/user-guide/101/normality-central-limit-theorem>“>„Central limit theorem and the normality assumption > Normality > Continuous distributions > Distribution > Statistical Reference Guide | Analyse-it® 6.15 documentation“. <i>analyse-it.com</i>. Retrieved 2024-05-17.</cite><li id=„cite_note-24“>^ <link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1215172403“ /><cite id=„CITEREFDEMİR2022“ class=„citation journal cs1“>DEMİR, Süleyman (2022-06-26). <a rel=„nofollow“ class=„external text“ href=„<https://dx.doi.org/10.21449/ijate.1101295>“>„Comparison of Normality Tests in Terms of Sample Sizes under Different Skewness and Kurtosis Coefficients“. <i>International Journal of Assessment Tools in Education</i>. 9 (2): 397–409. doi:<a rel=„nofollow“ class=„external text“ href=„<https://doi.org/10.21449%2Fijate.1101295>“>10.21449/ijate.1101295. ISSN <a rel=„nofollow“ class=„external text“ href=„<https://www.worldcat.org/issn/2148-7456>“>2148-7456.</cite><li id=„cite_note-25“><a href=„[Qgelm - <https://schnipsel.qgelm.de/>](https://en.wikipedia.org/wiki/Student%27s_t-
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href=„<https://www.youtube.com/watch?v=sajXLvfolmg&list=PLD15D38DC7AA3B737&index=2#t=13m31s>“>Econometrics lecture (topic: hypothesis testing) on YouTube by Mark Thoma<div class=„navbox-styles“><style data-mw-deduplicate=„TemplateStyles:r1129693374“ scoped=„scoped“></style><style data-mw-deduplicate=„TemplateStyles:r1228936124“ scoped=„scoped“></style></div><div role=„navigation“ class=„navbox“ aria-labelledby=„Statistics“><table class=„nowraplinks hlist mw-collapsible uncollapsed navbox-inner“><tbody><tr><th scope=„col“ class=„navbox-title“ colspan=„2“><link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1129693374“ /><style data-mw-deduplicate=„TemplateStyles:r1063604349“ scoped=„scoped“></style><div class=„navbar plainlinks hlist navbar-mini“><li class=„nv-view“><a href=„<https://en.wikipedia.org/wiki/Template:Statistics>“ title=„Template:Statistics“><abbr title=„View this template“>v</abbr><li class=„nv-talk“><abbr title=„Discuss this template“>t</abbr><li class=„nv-edit“><a href=„<https://en.wikipedia.org/wiki/Special:EditPage/Template:Statistics>“ title=„Special:EditPage/Template:Statistics“><abbr title=„Edit this template“>e</abbr></div><div id=„Statistics“><a href=„<https://en.wikipedia.org/wiki/Statistics>“ title=„Statistics“>Statistics</div></th></tr><tr><td class=„navbox-abovebelow“ colspan=„2“><div>OutlineIndex</div></td></tr><tr><td colspan=„2“ class=„navbox-list navbox-odd“><table class=„nowraplinks mw-collapsible mw-collapsed navbox-subgroup“><tbody><tr><th scope=„col“ class=„navbox-title“ colspan=„2“><div id=„Descriptive_statistics“>Descriptive statistics</div></th></tr><tr><td colspan=„2“ class=„navbox-list navbox-odd“><table class=„nowraplinks navbox-subgroup“><tbody><tr><th scope=„row“ class=„navbox-group“>Continuous data</th><td class=„navbox-list-with-group navbox-list navbox-odd“><table class=„nowraplinks navbox-subgroup“><tbody><tr><th scope=„row“ class=„navbox-group“>Center</th><td class=„navbox-list-with-group navbox-list navbox-odd“><div><a href=„<https://en.wikipedia.org/wiki/Mean>“ title=„Mean“>MeanArithmeticArithmetic-GeometricCubicGeneralized/powerGeometricHarmonicHeronianHeinzLehmer<a href=„<https://en.wikipedia.org/wiki/Median>“ title=„Median“>Median<a href=„[https://en.wikipedia.org/wiki/Mode_\(statistics\)](https://en.wikipedia.org/wiki/Mode_(statistics))“ title=„Mode

(statistics)">Mode</div></td></tr><tr><th scope="row" class="navbox-group">Dispersion</th><td class="navbox-list-with-group navbox-list navbox-even"><div>Average absolute deviationCoefficient of variationInterquartile rangePercentileRangeStandard deviationVariance</div></td></tr><tr><th scope="row" class="navbox-group">Shape</th><td class="navbox-list-with-group navbox-list navbox-odd"><div>Central limit theoremMomentsKurtosisL-momentsSkewness</div></td></tr><tr><th scope="row" class="navbox-group">Count data</th><td class="navbox-list-with-group navbox-list navbox-even"><div>Index of dispersion</div></td></tr><tr><th scope="row" class="navbox-group">Summary tables</th><td class="navbox-list-with-group navbox-list navbox-odd"><div>Contingency tableFrequency distributionGrouped data</div></td></tr><tr><th scope="row" class="navbox-group">Dependence</th><td class="navbox-list-with-group navbox-list navbox-even"><div>Partial correlationPearson product-moment correlationRank correlationKendall's τSpearman's ρScatter plot</div></td></tr><tr><th scope="row" class="navbox-group">Graphics</th><td class="navbox-list-with-group navbox-list navbox-odd"><div>Bar chartBiplot</div></td></tr></tbody></table></td></tr></table>

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[Control chart](#)

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[Run chart](#)

[Scatter plot](#)

[Stem-and-leaf display](#)

[Violin plot](#)

[Data collection](#)

[Design of experiments](#)

[Effect size](#)

[Missing data](#)

[Optimal design](#)

[Statistical population](#)

[Replication \(statistics\)](#)

[Sample size determination](#)

[Statistic](#)

[Statistical power](#)

[Survey methodology](#)

[Sampling \(statistics\)](#)

[Cluster sampling](#)

[Stratified sampling](#)

[Opinion poll](#)

[Questionnaire](#)

[Standard error](#)

[Experiment](#)

[Blocking \(statistics\)](#)

[Factorial experiment](#)

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parameter">ParameterlocationscaleshapeParametric familyLikelihood (monotone)Location–scale familyExponential familyCompletenessSufficiencyStatistical functionalBootstrap<a href=„<https://en.wikipedia.org/wiki/U-statistic>“ title=„U-statistic“>U<a href=„<https://en.wikipedia.org/wiki/V-statistic>“ title=„V-statistic“>VOptimal decisionloss functionEfficiencyStatistical distancedivergenceAsymptoticsRobustness</div></td></tr><tr><th scope=„row“ class=„navbox-group“>Frequentist inference</th><td class=„navbox-list-with-group navbox-list navbox-odd“><table class=„nowraplinks navbox-subgroup“><tbody><tr><th scope=„row“ class=„navbox-group“>Point estimation</th><td class=„navbox-list-with-group navbox-list navbox-even“><div>Estimating equationsMaximum likelihoodMethod of moments<a href=„<https://en.wikipedia.org/wiki/M-estimator>“ title=„M-estimator“>M-estimatorMinimum distanceUnbiased estimatorsMean-unbiased minimum-varianceRao–Blackwellization<a href=„<https://en.wikipedia.org/wiki/Blackwellization>“ title=„Blackwellization“>Blackwellization</div></td></tr></tbody></table></td></tr></table>

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multiplier<a href=„https://en.wikipedia.org/wiki/Wald_test“ title=„Wald
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test<a href=„<https://en.wikipedia.org/wiki/F-test>“ title=„F-test“><i>F</i>-
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squared<a href=„<https://en.wikipedia.org/wiki/G-test>“ title=„G-test“><i>G</i>-testKolmogorov–SmirnovAnderson–DarlingLillieforsJarque–BeraNormality (Shapiro–Wilk)Likelihood-ratio testModel selectionCross validationAICBIC</div></td></tr><tr><th scope=„row“ class=„navbox-group“>Rank statistics</th><td class=„navbox-list-with-group navbox-list navbox-even“><div>SignSample medianSigned rank (Wilcoxon)Hodges–Lehmann estimatorRank sum (Mann–Whitney)Nonparametric anova1-way (Kruskal–Wallis)2-way (Friedman)Ordered alternative (Jonckheere–Terpstra)Van der Waerden test</div></td></tr></tbody></table></td></tr><tr><th scope=„row“ class=„navbox-group“>Bayesian inference</th><td class=„navbox-list-with-group navbox-list navbox-odd“><div>Bayesian probabilitypriorposteriorCredible intervalBayes factorBayesian estimatorBayesian estimator</div></td></tr></tbody></table>

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analysis</div></th></tr><tr><td colspan=„2“ class=„navbox-list navbox-odd“><table
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model<a href=„https://en.wikipedia.org/wiki/Poisson_regression“ title=„Poisson
regression“>Log-linear modelMcNemar's
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href=„https://en.wikipedia.org/wiki/Cochran%27s_mantel%27s_haenszel_statistics“
title=„Cochran–Mantel–Haenszel
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statistics</div></td></tr><tr><th scope=„row“ class=„navbox-group“><a
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linear model">RegressionManovaPrincipal componentsCanonical correlationDiscriminant analysisCluster analysisClassificationStructural equation modelFactor analysisMultivariate distributionsElliptical distributionsNormal</div></td></tr><tr><th scope=„row“ class=„navbox-group“>Time-series</th><td class=„navbox-list-with-group navbox-list navbox-odd“><table class=„nowraplinks navbox-subgroup“><tbody><tr><th scope=„row“ class=„navbox-group“>General</th><td class=„navbox-list-with-group navbox-list navbox-odd“><div>DecompositionTrendStationaritySeasonal adjustmentExponential smoothing<a href=„<https://en.wikipedia.org/wiki/Cointegration>“ title=„Cointegration“>CointegrationStructural breakGranger causality</div></td></tr><tr><th scope=„row“ class=„navbox-group“>Specific tests</th><td class=„navbox-list-with-group navbox-list navbox-even“><div>Dickey–FullerJohansenQ-statistic (Ljung–Box)Durbin–WatsonBreusch–Godfrey</div></td></tr><tr><th scope=„row“ class=„navbox-group“>Time domain</th><td class=„navbox-list-with-group navbox-list navbox-odd“><div><a href=„<https://en.wikipedia.org/wiki/Autocorrelation>“

title=„Autocorrelation“>Autocorrelation (ACF)partial (PACF)Cross-correlation (XCF)ARMA modelARIMA model (Box–Jenkins)Autoregressive conditional heteroskedasticity (ARCH)Vector autoregression (VAR)</div></td></tr><tr><th scope=„row“ class=„navbox-group“>Frequency domain</th><td class=„navbox-list-with-group navbox-list navbox-even“><div>Spectral density estimationFourier analysisLeast-squares spectral analysisWaveletWhittle likelihood</div></td></tr></tbody></table></td></tr><tr><th scope=„row“ class=„navbox-group“>Survival</th><td class=„navbox-list-with-group navbox-list navbox-odd“><table class=„nowraplinks navbox-subgroup“><tbody><tr><th scope=„row“ class=„navbox-group“>Survival function</th><td class=„navbox-list-with-group navbox-list navbox-odd“><div>Kaplan–Meier estimator (product limit)Proportional hazards modelsAccelerated failure time (AFT) modelFirst hitting time</div></td></tr><tr><th scope=„row“ class=„navbox-group“>Hazard function</th><td class=„navbox-list-with-group navbox-list navbox-even“><div>Nelson–Aalen estimator</div></td></tr><tr><th scope=„row“ class=„navbox-group“>Test</th><td class=„navbox-list-with-group navbox-list navbox-odd“><div>Log-rank test</div></td></tr></tbody></table></td></tr></tbody></table></td></tr><tr><td colspan=„2“ class=„navbox-list navbox-odd“><table class=„nowraplinks mw-collapsible mw-collapsed navbox-subgroup“><tbody><tr><th scope=„col“ class=„navbox-title“ colspan=„2“><div id=„Applications“>Applications</div></th></tr><tr><td colspan=„2“ class=„navbox-list navbox-odd“><table class=„nowraplinks navbox-subgroup“><tbody><tr><th scope=„row“

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upload.wikimedia.org/wikipedia/commons/thumb/3/37/People_icon.svg/32px-People_icon.svg.png 2x“ data-file-width=„100“ data-file-height=„100“ referrerpolicy=„no-referrer“ />WikiProject</div></td></tr></tbody></table></div><div class=„navbox-styles“><link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1129693374“ /><link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1228936124“ /></div><div role=„navigation“ class=„navbox“ aria-labelledby=„Public_health“><table class=„nowraplinks hlist mw-collapsible autocollapse navbox-inner“><tbody><tr><th scope=„col“ class=„navbox-title“ colspan=„2“><link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1129693374“ /><link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1063604349“ /><div class=„navbar plainlinks hlist navbar-mini“><li class=„nv-view“><abbr title=„View this template“>v</abbr><li class=„nv-talk“><abbr title=„Discuss this template“>t</abbr><li class=„nv-edit“><abbr title=„Edit this template“>e</abbr></div><div id=„Public_health“>Public

health

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Auxology Biological hazard Chief Medical Officer Cultural competence in health care Deviance (sociology) Environmental health Euthenics Genomics Globalization and disease Harm reduction Health economics Health literacy Health policy Health system Health care reform Management of depression Public health law National public health institute Health politics Maternal health Medical anthropology Medical sociology Mental health Ministers Pharmaceutical policy Pollution Air Water Soil Radiation Light Public health intervention Public health laboratory

href=„https://en.wikipedia.org/wiki/Sexual_and_reproductive_health“ title=„Sexual and reproductive health“>Sexual and reproductive healthSocial psychologySociology of health and illness</div></td></tr><tr><th scope=„row“ class=„navbox-group“>Preventive healthcare</th><td class=„navbox-list-with-group navbox-list navbox-even“><div>Behavior changeTheoriesFamily planningHealth promotionHuman nutritionHealthy dietPreventive nutrition<a href=„<https://en.wikipedia.org/wiki/Hygiene>“ title=„Hygiene“>HygieneFood safetyHand washingInfection controlOral hygieneOccupational safety and healthHuman factors and ergonomicsOccupational hygieneControlled DrugsInjury preventionMedicineNursingPatient safetyOrganization<a href=„<https://en.wikipedia.org/wiki/Pharmacovigilance>“ title=„Pharmacovigilance“>PharmacovigilanceSafe sex<a href=„<https://en.wikipedia.org/wiki/Sanitation>“ title=„Sanitation“>SanitationEmergency sanitationFecal–oral transmissionOpen defecationSanitary sewer<a href=„https://en.wikipedia.org/wiki/Waterborne_diseases“

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safety management</th><td class=„navbox-list-with-group navbox-list navbox-even“><div>FoodAdditiveChemistryEngineeringMicrobiologyProcessingSafetySafety scandalsGenetically modified foodGood agricultural practiceGood manufacturing practice<a href=„<https://en.wikipedia.org/wiki/HACCP>“ title=„HACCP“>HACCPISO 22000</div></td></tr><tr><th scope=„row“ class=„navbox-group“>Health behavioral
sciences</th><td class=„navbox-list-with-group navbox-list navbox-odd“><div>Diffusion of innovationsHealth belief modelHealth communicationHealth

psychology

- [Positive deviance](https://en.wikipedia.org/wiki/Positive_deviance "Positive deviance")
- [PRECEDE–PROCEED model](https://en.wikipedia.org/wiki/PRECEDE%20%93PROCEED_model "PRECEDE–PROCEED model")
- [Social cognitive theory](https://en.wikipedia.org/wiki/Social_cognitive_theory "Social cognitive theory")
- [Social norms approach](https://en.wikipedia.org/wiki/Social_norms_approach "Social norms approach")
- [Theory of planned behavior](https://en.wikipedia.org/wiki/Theory_of_planned_behavior "Theory of planned behavior")
- [Transtheoretical model](https://en.wikipedia.org/wiki/Transtheoretical_model "Transtheoretical model")

scope="row"
Organizations, education and history
Caribbean
Caribbean Public Health Agency
China
Center for Disease Control and Prevention
Europe
Centre for Disease Prevention and Control
Committee on the Environment, Public Health and Food Safety
India
Ministry of Health and Family Welfare
Canada
Health Canada
Public Health Agency
U.S.
Centers for Disease Control and Prevention
Health departments in the United States
Council on Education for Public Health
Public Health Service
World Health Organization
World Toilet Organization
Full list
Health education
Higher education

href=„https://en.wikipedia.org/wiki/Bachelor_of_Science_in_Public_Health“ title=„Bachelor of Science in Public Health“>Bachelor of Science in Public HealthDoctor of Public HealthProfessional degrees of public healthSchools of public health</div></td></tr><tr><th scope=„row“ class=„navbox-group“>History</th><td class=„navbox-list-with-group navbox-list navbox-even“><div>Sara Josephine BakerSamuel Jay CrumbineCarl Rogers DarnallJoseph ListerMargaret SangerJohn SnowTyphoid MaryRadium GirlsGerm theory of diseaseSocial hygiene movement</div></td></tr></tbody></table></td></tr><tr><td class=„navbox-abovebelow“ colspan=„2“><div>Category<img alt=„“ src=„<https://upload.wikimedia.org/wikipedia/en/thumb/4/4a/ Commons-logo.svg/12px- Commons-logo.svg.png>“ width=„12“ height=„16“ class=„mw-file-element“ srcset=„upload.wikimedia.org/wikipedia/en/thumb/4/4a/ Commons-logo.svg/18px- Commons-logo.svg.png 1.5x, upload.wikimedia.org/wikipedia/en/thumb/4/4a/ Commons-logo.svg/24px- Commons-logo.svg.png 2x“ data-file-width=„1024“ data-file-height=„1376“ referrerpolicy=„no-referrer“ />CommonsWikiProject</div></td></tr></tbody></table></div><div class=„navbox-styles“><link rel=„mw-deduplicated-inline-style“ href=„denied:mw-

data:TemplateStyles:r1129693374" /><link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1228936124" /></div><div role=„navigation“ class=„navbox authority-control“ aria-label=„Navbox“><table class=„nowraplinks hlist navbox-inner“><tbody><tr><th scope=„row“ class=„navbox-group“>Authority control databases: National </th><td class=„navbox-list-with-group navbox-list navbox-odd“><div>Germany</div></td></tr></tbody></table></div></dd></dl></div><noscript></noscript><div class=„printfooter“ data-nosnippet=„“>Retrieved from „https://en.wikipedia.org/w/index.php?title=Student%27s_t-test&oldid=1224247091“</div></div> </html>

1)

$\mu_1, \dots, \mu_n, 0, \dots, 0)^T, (\sigma_1^2, \dots, \sigma_n^2)^T$

referrerpolicy=„no-referrer“ /></dd></dl><p>From the above distribution we see that</p><dl><dd></dd></dl><dl><dd></dd></dl><dl><dd></dd></dl><dl><dd></dd></dl><dl><dd></dd></dl><h3>Dependent <i>t</i>-test for paired samples<a href=„https://en.wikipedia.org/w/index.php?title=Student%27s_t-test&action=edit§ion“

=15" title="Edit section: Dependent t-test for paired samples">edit]</h3><p>This test is used when the samples are dependent; that is, when there is only one sample that has been tested twice (repeated measures) or when there are two samples that have been matched or „paired“. This is an example of a paired difference test. The t statistic is calculated as</p><dl><dd></dd></dl><p>where and are the average and standard deviation of the differences between all pairs. The pairs are e.g. either one person's pre-test and post-test scores or between-pairs of persons matched into meaningful groups (for instance, drawn from the same family or age group: see table). The constant μ_0 is zero if we want to test whether the average of the difference is significantly different. The degree of freedom used is $n-1$, where n represents the number of pairs.</p><dl><dd><table><tbody><tr><th>Pair</th><th>Name</th><th>Age</th><th>Test</th></tr><tr><td>1</td><td>John</td><td>35</td><td>250</td></tr><tr><td>1</td><td>Jane</td><td>36</td><td>340</td></tr><tr><td>2</td><td>Jimmy</td><td>22</td><td>460</td></tr><tr><td>2</td><td>Jessy</td><td>21</td><td>200</td></tr></tbody></table></dd><td><table class="wikitable" style="text-align: right;"><caption>Example of repeated measures</caption><tbody><tr><th>Number</th><th>Name</th><th>Test 1</th><th>Test 2</th></tr><tr><td>1</td><td>Mike</td><td>35%</td><td>67%</td></tr><tr><td>2</td><td>Melanie</td><td>50%</td><td>46%</td></tr><tr><td>3</td><td>Melissa</td><td>90%</td><td>86%</td></tr><tr><td>4</td><td>Mitchell</td><td>78%</td><td>91%</td></tr></tbody></table></td></tr></tbody></table><h2>Worked examples[edit]</h2><style data-mw-deduplicate="TemplateStyles:r1097763485" scoped="scoped"></style><p>Let A_1 denote a set obtained by drawing a random sample of six measurements:</p><dl><dd></dd></dl><p>and let A_2 denote a second set obtained similarly:</p><dl><dd></dd></dl><p>These could be, for example, the weights of screws that were manufactured by two different machines.</p><p>We will carry out tests of the null hypothesis that the means of the populations from which the two samples were taken are equal.</p><p>The difference between the two sample means, each denoted by $\bar{X}_i$, which appears in the numerator for all the two-sample testing approaches discussed above, is</p><dl><dd></dd></dl>

4d88f41" class="mwe-math-fallback-image-inline mw-invert skin-invert" aria-hidden="true" alt="{\displaystyle {\bar {X}}_{1}-{\bar {X}}_{2}=0.095.}" referrerpolicy="no-referrer" /></dd></dl><p>The sample standard deviations for the two samples are approximately 0.05 and 0.11, respectively. For such small samples, a test of equality between the two population variances would not be very powerful. Since the sample sizes are equal, the two forms of the two-sample <i>t</i>-test will perform similarly in this example.</p><h3>Unequal variances[edit]</h3><p>If the approach for unequal variances (discussed above) is followed, the results are</p><dl><dd></dd></dl><p>and the degrees of freedom</p><dl><dd></dd></dl><p>The test statistic is approximately 1.959, which gives a two-tailed test <i>p</i>-value of 0.09077.</p><h3>Equal variances[edit]</h3><p>If the approach for equal variances (discussed above) is followed, the results are</p><dl><dd></dd></dl><p>and the degrees of freedom</p><dl><dd></dd></dl><p>The test statistic is approximately equal to 1.959, which gives a two-tailed <i>p</i>-value of 0.07857.</p><h2>Related statistical tests[edit]</h2><h3>Alternatives to the <i>t</i>-test for location problems[edit]</h3><p>The <i>t</i>-test provides an exact test for the equality of the means of two i.i.d. normal populations with unknown, but equal, variances. (Welch's <i>t</i>-test is a nearly exact test for the case where the data are normal but the variances may differ.) For moderately large samples and a one tailed test, the <i>t</i>-test is relatively robust to moderate violations of the normality assumption.^{[26]} In large enough samples, the t-test asymptotically approaches the <i>z</i>-test, and becomes robust even to large deviations from normality.^{[19]}</p><p>If the data are substantially non-normal and the sample size is small, the <i>t</i>-test can give misleading results. See Location test for Gaussian scale

mixture distributions for some theory related to one particular family of non-normal distributions.

When the normality assumption does not hold, a [non-parametric](https://en.wikipedia.org/wiki/Non-parametric "Non-parametric") alternative to the *t*-test may have better [statistical power](https://en.wikipedia.org/wiki/Statistical_power "Statistical power"). However, when data are non-normal with differing variances between groups, a *t*-test may have better [type-1 error](https://en.wikipedia.org/wiki/Type_1_error "Type 1 error") control than some non-parametric alternatives.^{[\[27\]](https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-27 "cite_ref-27")} Furthermore, non-parametric methods, such as the [Mann–Whitney U test](https://en.wikipedia.org/wiki/Mann%E2%80%93Whitney_U_test "Mann–Whitney U test") discussed below, typically do not test for a difference of means, so should be used carefully if a difference of means is of primary scientific interest.^{[\[19\]](https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-0-19 "cite_ref-0_19-2")} For example, Mann–Whitney U test will keep the type 1 error at the desired level α if both groups have the same distribution. It will also have power in detecting an alternative by which group B has the same distribution as A but after some shift by a constant (in which case there would indeed be a difference in the means of the two groups). However, there could be cases where group A and B will have different distributions but with the same means (such as two distributions, one with positive skewness and the other with a negative one, but shifted so to have the same means). In such cases, MW could have more than α level power in rejecting the Null hypothesis but attributing the interpretation of difference in means to such a result would be incorrect.

In the presence of an [outlier](https://en.wikipedia.org/wiki/Outlier "Outlier"), the *t*-test is not robust. For example, for two independent samples when the data distributions are asymmetric (that is, the distributions are [skewed](https://en.wikipedia.org/wiki/Skewness "Skewness")) or the distributions have large tails, then the Wilcoxon rank-sum test (also known as the [Mann–Whitney *U* test](https://en.wikipedia.org/wiki/Mann%E2%80%93Whitney_U_test "Mann–Whitney U test")) can have three to four times higher power than the *t*-test.^{[\[26\]](https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-Sawilowsky-Blair_26-1 "cite_ref-Sawilowsky-Blair_26-1")}^{[\[28\]](https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-28 "cite_ref-28")}^{[\[29\]](https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-29 "cite_ref-29")} The nonparametric counterpart to the paired samples *t*-test is the [Wilcoxon signed-rank test](https://en.wikipedia.org/wiki/Wilcoxon_signed-rank_test "Wilcoxon signed-rank test") for paired samples. For a discussion on choosing between the *t*-test and nonparametric alternatives, see Lumley, et al. (2002).^{[\[19\]](https://en.wikipedia.org/wiki/Student%27s_t-test#cite_note-0-19 "cite_ref-0_19-3")}

One-way [analysis of variance](https://en.wikipedia.org/wiki/Analysis_of_variance "Analysis of variance") (ANOVA) generalizes the two-sample *t*-test when the data belong to more than two groups.

A design which includes both paired observations and independent observations

When both paired observations and independent observations are present in the two sample design, assuming data are missing completely at random (MCAR), the paired observations or independent observations may be discarded in order to proceed with the

standard tests above. Alternatively making use of all of the available data, assuming normality and MCAR, the generalized partially overlapping samples t-test could be used.

^{id=„cite_ref-Partover_30-0“ class=„reference“>[30]}</p><h3>Multivariate testing[edit]</h3><link rel=„mw-deduplicated-inline-style“ href=„denied:mw-data:TemplateStyles:r1033289096“ /><div role=„note“ class=„hatnote navigation-not-searchable“>Main article: Hotelling's T-squared distribution</div><p>A generalization of Student's <i>t</i> statistic, called Hotelling's <i>t</i>-squared statistic, allows for the testing of hypotheses on multiple (often correlated) measures within the same sample. For instance, a researcher might submit a number of subjects to a personality test consisting of multiple personality scales (e.g. the Minnesota Multiphasic Personality Inventory). Because measures of this type are usually positively correlated, it is not advisable to conduct separate univariate <i>t</i>-tests to test hypotheses, as these would neglect the covariance among measures and inflate the chance of falsely rejecting at least one hypothesis (Type I error). In this case a single multivariate test is preferable for hypothesis testing. Fisher's Method for combining multiple tests with <i>alpha</i> reduced for positive correlation among tests is one. Another is Hotelling's <i>T</i>² statistic follows a <i>T</i> distribution. However, in practice the distribution is rarely used, since tabulated values for <i>T</i>² are hard to find. Usually, <i>T</i>² is converted instead to an <i>F</i> statistic.</p><p>For a one-sample multivariate test, the hypothesis is that the mean vector (μ) is equal to a given vector (μ₀). The test statistic is Hotelling's <i>t</i>²:</p><dl><dd><img src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/a4d0dec2d43168c2a110e5e9b2a97e80f8337059“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„{\displaystyle t^{2}=n({\bar {\mathbf {x} }}-{\boldsymbol {\mu }}_{0})'({\mathbf {S} }^{-1}({\bar {\mathbf {x} }}-{\boldsymbol {\mu }}_{0}))}</math>“ referrerpolicy=„no-referrer“ /></dd></dl><p>where <i>n</i> is the sample size, x is the vector of column means and S is an <i>m</i> × <i>m</i> sample covariance matrix.</p><p>For a two-sample multivariate test, the hypothesis is that the mean vectors (μ₁, μ₂) of two samples are equal. The test statistic is Hotelling's two-sample <i>t</i>²:</p><dl><dd><img src=„https://wikimedia.org/api/rest_v1/media/math/render/svg/71063bdcd2da01eec91c537247dc3413e9f2e08f“ class=„mwe-math-fallback-image-inline mw-invert skin-invert“ aria-hidden=„true“ alt=„{\displaystyle t^{2}={\frac {n_{1}n_{2}}{n_{1}+n_{2}}}\left({\bar {\mathbf {x} }}_{1}-{\bar {\mathbf {x} }}_{2}\right)'{\mathbf {S} }_{\text{pooled}}^{-1}\left({\bar {\mathbf {x} }}_{1}-{\bar {\mathbf {x} }}_{2}\right).}</math>“ referrerpolicy=„no-referrer“ /></dd></dl><h3>The</sup></sup></sup>

two-sample t-test is a special case of simple linear regression[edit]</h3><p>The two-sample t-test is a special case of simple linear regression as illustrated by the following example.</p><p>A clinical trial examines 6 patients given drug or placebo. Three (3) patients get 0 units of drug (the placebo group). Three (3) patients get 1 unit of drug (the active treatment group). At the end of treatment, the researchers measure the change from baseline in the number of words that each patient can recall in a memory test.</p><p></p><p>A table of the patients' word recall and drug dose values are shown below. </p><table class=„wikitable“><tbody><tr><th>Patient</th><th>drug.dose</th><th>word.recall</th></tr><tr><th>1</th><th>0</th><td>1</td></tr><tr><th>2</th><th>0</th><td>2</td></tr><tr><th>3</th><th>0</th><td>3</td></tr><tr><th>4</th><th>1</th><td>5</td></tr><tr><th>5</th><th>1</th><td>6</td></tr><tr><th>6</th><th>1</th><td>7</td></tr></tbody></table><p>Data and code are given for the analysis using the R programming language with the

t.test

and

lm

functions for the t-test and linear regression. Here are the same (fictitious) data above generated in R.</p><div class=„mw-highlight mw-highlight-lang-r mw-content-ltr“ dir=„ltr“><pre>>word.recall.data=data.frame(drug.dose=c(0,0,0,1,1,1),word.recall=c(1,2,3,5,6,7

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